

# $\sigma$ CDM coupled to radiation. Dark energy and Universe acceleration

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**Abstract.** Recently the Chiral Cosmological Model (CCM) coupled to cold dark matter (CDM) has been investigated as  $\sigma$ CDM model to study the observed accelerated expansion of the Universe. Dark sector fields (as Dark Energy content) coupled to cosmic dust were considered as the source of Einstein gravity in Friedmann-Robertson-Walker (FRW) cosmology. Such model had a beginning at the matter-dominated era. The purposes of our present investigation are two folds: to extend «life» of the  $\sigma$ CDM for earlier times to radiation-dominated era and to take into account variation of the exponential potential  $V = V_0 \exp\left(-\sqrt{\lambda} \frac{\varphi}{M_{Pl}}\right) + V_0 \exp\left(-\sqrt{\lambda} \frac{\chi}{M_{Pl}}\right)$  via variation of the interaction parameter  $\lambda$ .

We use Markov Chain Monte Carlo (MCMC) procedure to investigate possible values of initial conditions constrained by the measured amount of the dark matter, dark energy and radiation component today.

Our analysis includes dark energy contribution to critical density, the ratio of the kinetic and potential energies, deceleration parameter, effective equation of state and evolution of DE equation of state with variation of coupling constant  $\lambda$ . A comparison with the  $\Lambda$ CDM model was performed. A new feature of the model is the existence of some values of potential coupling constant, leading to a  $\sigma$ CDM solution without transit into accelerated expansion epoch.

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## Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Extension of <math>\sigma</math>CDM model to radiation dominated era</b>	<b>2</b>
<b>3</b>	<b>Constraining and solving <math>\sigma</math>CDM equations</b>	<b>5</b>
<b>4</b>	<b>Discussion</b>	<b>6</b>
<b>5</b>	<b>Conclusion and future work</b>	<b>7</b>

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## 1 Introduction

To explain the present accelerated expansion of the universe, confirmed by a range of astrophysical observations, one may use dark energy (in a wide sense as the source of the acceleration with negative pressure) or one has to modify Einstein's gravitation theory.

The simplest and very attractive model of dark energy is one based on the cosmological constant  $\Lambda$  as a repulsive force [1–5]. The resulting  $\Lambda$ CDM model with cold dark matter component is consistent with astrophysical observations in many respects. However, the model is faced with two problems: fine tuning and cosmic coincidence.

The cosmological constant problem essentially has to do with our (mis)understanding of the nature of gravity [6]. Other option in understanding of cosmic acceleration and dark energy is the infrared modification of general relativity that may be responsible for the large-scale behavior of the universe [7].

Also today a very popular approach concerns modified gravity, where additional degrees of freedom must screen themselves from local tests of gravity. So there exist screening mechanisms associated with chameleon and galileon mechanisms, as well as massive gravity and Vainshtein mechanisms (see, for details [8]). Various observational and experimental tests have been reflected in this work as well.

Another approach to avoid fine tuning and cosmic coincidence problems is connected with a variety of scalar field models, including quintessence, phantom's and tachyon's fields etc. have been proposed. One of such model, so called  $\phi$ CDM model, is presented in the work Park et. al. [9]. The k-essence model providing a dynamical solution for explaining naturally why the universe has entered an epoch of accelerated expansion at a late stage of its evolution also essentially relies on using scalar fields. The solution that proposed in k-essence model avoids fine-tuning of parameters and anthropic arguments [10].

In the present article we extend in some sense such types of models to a multiplet of the scalar fields and work with the two-component chiral cosmological model in details.

A chiral cosmological model (CCM) have been proposed as a non-linear sigma model with the potential of (self)interaction and implemented into cosmology (for review, see [11]). Application of a CCM coupled to a cold dark matter for later time inflation leads to proposal of a new type of models with quintessence and quintom fields, dubbed as  $\sigma$ CDM models [12]. Also the cosmological constant problem is absent in CCM as there exist a dynamic potential interaction in CCM. In this work the necessity of extension of this model to more general CCM was mentioned to obtain better agreement with observational data. On this

way we investigate now an extension of  $\sigma$ CDM models presented in [12]. Namely we start from  $\sigma$ CDM model that more drastically differs from  $\Lambda$ CDM. To go deeper back in time we include radiation in the model action. Thus our consideration may start from radiation dominated era at times which roughly corresponds to nucleosynthesis stage. In our approach we present dark energy as kinetically and potentially interacting two scalar fields described by a chiral cosmological model. We also include a coupling to perfect fluids, cold dark matter and radiation.

Recently authors of [13] investigated multiple scalar fields model, including a model of two canonical fields with coupled exponential potentials arising in string theory. The model they investigated is similar to our one, but devoted to cosmological scaling solutions.

It is also very common to study dark energy models by means of some parametrization of the dark energy equation of state. One often used dark energy parametrization of dark energy is CPL parametrization with dependence on scale factor  $a(t)$  in the form  $w(a) = w_0 + w_a(1 - a)$  [14] (called CPL after Chevallier – Polarski – Linder). The authors [14] mentioned that almost all the cosmological constraints on dark energy are based on this parametrization. The question still stands whether there are possible dark energy evolutions that one misses using the CPL parametrization. Their results motivate the construction of models of dark energy which lead to phantom behavior. This means going beyond standard possibilities for dark energy involving a scalar field with a positive kinetic energy term only which do not lead to a violation of the weak energy condition [14].

There exist also the Scherrer and Sen parametrization [15] represented by a rather complicated formula with dependence  $w(a) = (1 + w_0)f(\Omega_{DE}, a)$  with a certain function  $f$  and the Generalized Chaplygin Gas (GCG) parametrization presented by the relation  $w(a) = -\frac{A}{A + (1-A)a^{-3(1+\alpha)}}$ ,  $0 \geq \alpha \geq 1$ . Thus we can see that equation of state (EoS) may not be constant and could vary with the scale factor  $a$ .

In our presentation the EoS parameter is obtained numerically, and the dependence on  $a$  is displayed graphically.

Authors [16] attracted attention to the tension between measured the Hubble constant  $H_0$  by the Planck collaboration and by the several direct probes on  $H_0$ . To avoid this tension they found out that EoS  $w$  should be less the  $-1$  or it should be time-evolving. Also they mentioned that with such tension the concordance cosmological model ( $\Lambda$ CDM) is in fact incomplete. This is also one more reason to analyze the  $\sigma$ CDM model.

The general plan of the paper is the following. In the sec. 2 we represent the main equations of the  $\sigma$ CDM model under investigation and write down basic cosmological quantities that can be extracted from that equations. Also we make a choice of new variables, more appropriate in the approach connected with usage of initial values of quantities. In sec. 3 we describe observational constraints on  $\sigma$ CDM which we use here and the MCMC procedure. In sec. 4 we discuss the dependence of cosmological dynamics on varied potential interaction parameters values  $\lambda$ , make comparison with  $\Lambda$ CDM model, as well as point out essential features of  $\sigma$ CDM model, such as magnitude of kinetic interaction with relation to potential one. Sec. 5 is devoted for possible extension of the work we have done in this paper.

## 2 Extension of $\sigma$ CDM model to radiation dominated era

We start with consideration of a Chiral Cosmological Model (CCM) coupled to perfect fluid. Such a model related to (dark) matter source was called  $\sigma$ CDM model [12, 17].

Our intention now is to extend the model deeper back on time to the end of BB nucleosynthesis and beginning of the radiation era.

The action of CCM coupled to perfect fluid is

$$S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} g^{\mu\nu} h_{AB} \partial_\mu \varphi^A \partial_\nu \varphi^B - V(\varphi^A) \right) + S_{pf},$$

where  $S_{pf}$  accounts for perfect fluid (cold dark matter). We choose chiral metric components as  $h_{11} = 1$ ,  $h_{22} = \exp\left(\sqrt{\mu} \frac{\varphi - \varphi_i}{M_{Pl}}\right)$  and the (self)action potential in the form  $V = V_0 \exp\left(-\sqrt{\lambda} \frac{\varphi}{M_{Pl}}\right) + V_0 \exp\left(-\sqrt{\lambda} \frac{\chi}{M_{Pl}}\right)$ , where  $M_{Pl} \equiv 1/\sqrt{8\pi G}$ . Also we have to include in the model (dark and baryon) matter and radiation with dynamics represented by equations of energy-momentum conservation for each component.

To describe the dynamics of the model we study the system of Friedmann and chiral cosmological fields equations:

$$H^2 = \frac{8\pi G}{3} \left[ \rho_m + \rho_r + \frac{1}{2} \dot{\varphi}^2 + \frac{1}{2} h_{22} \dot{\chi}^2 + V(\varphi, \chi) \right], \quad (2.1)$$

$$\ddot{\varphi} + 3H\dot{\varphi} - \frac{1}{2} \frac{\partial h_{22}}{\partial \varphi} \dot{\chi}^2 + \frac{\partial V}{\partial \varphi} = 0, \quad (2.2)$$

$$\ddot{\chi} + 3H\dot{\chi} + \frac{1}{h_{22}} \frac{\partial h_{22}}{\partial \varphi} \dot{\varphi} \dot{\chi} + \frac{1}{h_{22}} \frac{\partial V}{\partial \chi} = 0. \quad (2.3)$$

Let us mention here that the second Einstein's equation (Raichaudhury equation) can also be derived by using linear combination of chiral cosmological fields, conservation of matter and radiation, and the Friedmann equation.

The key idea [18], that allows us to solve this ODE system, is to rely on initial values of quantities presented there. It means, that energy-momentum tensor conservation gives us energy densities of matter and radiation components in the following form

$$\rho_r = \rho_r^{(i)} \left( \frac{a}{a_i} \right)^{-4} = \frac{3H_i^2}{8\pi G} \Omega_r^{(i)} \left( \frac{a}{a_i} \right)^{-4},$$

$$\rho_m = \rho_m^{(i)} \left( \frac{a}{a_i} \right)^{-3} = \frac{3H_i^2}{8\pi G} \Omega_m^{(i)} \left( \frac{a}{a_i} \right)^{-3},$$

$$\rho_c^{(i)} = \frac{3H_i^2}{8\pi G},$$

where we introduced the critical density of the Universe for some initial moment of time which corresponds to scale factor value  $a = a_i$ . This approach allows us besides to get a feeling about values of initial amounts of dark energy, matter and radiation. We can estimate these values from the  $\Lambda$ CDM model if we put in corresponding expressions for the amounts of dark energy, matter and radiation today. The latter values should be in concordance with modern observational data, therefore we use values  $\Omega_{DE}^{(0)} \approx 0.7$ ,  $\Omega_m^{(0)} \approx 0.3$ ,  $\Omega_r^{(0)} = 5 \cdot 10^{-5}$ .

There is a restriction for maximum possible amount of dark energy (or in general scalar field model contribution) for nucleosynthesis epoch  $\Omega_{DE} < 0.045$  [9, 19, 20]. We take this value as upper bound for dark energy contribution at initial time. In our analysis we want to

be rather conservative and let lower bound of the dark energy amount be equal to  $1 \cdot 10^{-25}$ . These values are used for boundary values of  $\Omega_{DE}^{(i)}$  in MCMC procedure.

According to our choice above we would like to use new type of dimensionless variables [18]

$$s = \frac{a}{a_i}, \quad y = \frac{\varphi - \varphi_i}{M_P}, \quad z = \frac{\chi - \chi_i}{M_P}.$$

$$x = H_i(t - t_i), \quad \langle \rangle = \frac{d}{dx}, \quad M_P^2 = \frac{1}{8\pi G},$$

$$\bar{V} = V_0 M_P^{-2} H_i^{-2} \exp\left(-\sqrt{\lambda} M_P^{-1} \varphi_i\right) = V_0 M_P^{-2} H_i^{-2} \exp\left(-\sqrt{\lambda} M_P^{-1} \chi_i\right).$$

Thus the original system of equations is transformed to

$$\left(\frac{s'}{s}\right)^2 = \Omega_m^{(i)} s^{-3} + \Omega_r^{(i)} s^{-4} + \frac{1}{3} \left[ \frac{1}{2} y'^2 + \frac{1}{2} h_{22} z'^2 + \bar{V} e^{-\sqrt{\lambda} y} + \bar{V} e^{-\sqrt{\lambda} z} \right], \quad (2.4)$$

$$y'' + 3H y' - \frac{1}{2} \sqrt{\mu} e^{\sqrt{\mu} y} z'^2 - \sqrt{\lambda} \bar{V} e^{-\sqrt{\lambda} y} = 0, \quad (2.5)$$

$$z'' + 3H z' + \sqrt{\mu} e^{\sqrt{\mu} y} y' z' - \sqrt{\lambda} e^{\sqrt{\mu} y} \bar{V} e^{-\sqrt{\lambda} z} = 0. \quad (2.6)$$

The chiral cosmological fields act as dark energy and contributions to the critical density of the universe and the dark energy equation of state parameter as

$$\Omega_{DE} = \frac{\frac{1}{3} \left[ \frac{1}{2} y'^2 + \frac{1}{2} h_{22} z'^2 + \bar{V} e^{-\sqrt{\lambda} y} + \bar{V} e^{-\sqrt{\lambda} z} \right]}{\Omega_m^{(i)} s^{-3} + \Omega_r^{(i)} s^{-4} + \frac{1}{3} \left[ \frac{1}{2} y'^2 + \frac{1}{2} h_{22} z'^2 + \bar{V} e^{-\sqrt{\lambda} y} + \bar{V} e^{-\sqrt{\lambda} z} \right]}, \quad (2.7)$$

$$\omega_{DE} = \frac{\frac{1}{2} y'^2 + \frac{1}{2} h_{22} z'^2 - \bar{V} e^{-\sqrt{\lambda} y} - \bar{V} e^{-\sqrt{\lambda} z}}{\frac{1}{2} y'^2 + \frac{1}{2} h_{22} z'^2 + \bar{V} e^{-\sqrt{\lambda} y} + \bar{V} e^{-\sqrt{\lambda} z}}. \quad (2.8)$$

Initial conditions for chiral cosmological fields are

$$y'_i = z'_i = \sqrt{\frac{3}{2} \left( 1 - \Omega_m^{(i)} - \Omega_r^{(i)} \right) \left( 1 - \omega_{DE}^{(i)} \right)},$$

and for the potential is

$$\bar{V} = \frac{3}{4} \left( 1 - \Omega_m^{(i)} - \Omega_r^{(i)} \right) \left( 1 - \omega_{DE}^{(i)} \right),$$

It needs to keep in mind that we consider initial contributions of the both chiral cosmological fields to be equal  $\Omega_\varphi^{(i)} = \Omega_\chi^{(i)}$ .

In the present paper we analyze the evolution of such quantities as deceleration parameter, effective equation of state parameter and chiral cosmological fields kinetic and potential energies ratio. We also make a comparison with the corresponding quantities of the  $\Lambda$ CDM model. Let us write down expressions for them in terms of set of variables we introduced previously

$$\begin{aligned}
q^{\sigma\text{CDM}} &= -\frac{\ddot{a}}{\left(\frac{\dot{a}}{a}\right)^2} = \frac{\frac{4\pi G}{3} \sum_{\alpha} (\rho_{\alpha} + 3p_{\alpha})}{\frac{8\pi G}{3} \sum_{\alpha} \rho_{\alpha}} = \frac{1}{2} \frac{[\rho_r + \rho_m + \rho_{\sigma} + 3p_r + 3p_m + 3p_{\sigma}]}{[\rho_r + \rho_m + \rho_{\sigma}]} = \\
&= \frac{1}{2} \frac{\left[2\Omega_r^{(i)} s^{-4} + \Omega_m^{(i)} s^{-3} + \frac{1}{3} \left[2y'^2 + 2h_{22}z'^2 - 2\bar{V}e^{-\sqrt{\lambda}y} - 2\bar{V}e^{-\sqrt{\lambda}z}\right]\right]}{\left[\Omega_r^{(i)} s^{-4} + \Omega_m^{(i)} s^{-3} + \frac{1}{3} \left[\frac{1}{2}y'^2 + \frac{1}{2}h_{22}z'^2 + \bar{V}e^{-\sqrt{\lambda}y} + \bar{V}e^{-\sqrt{\lambda}z}\right]\right]}, \quad (2.9)
\end{aligned}$$

$$\begin{aligned}
\omega_{eff}^{\sigma\text{CDM}} &= \frac{\sum_{\alpha} p_{\alpha}}{\sum_{\alpha} \rho_{\alpha}} = \frac{p_r + p_m + p_{\sigma}}{\rho_r + \rho_m + \rho_{\sigma}} = \\
&= \frac{\frac{1}{3}\Omega_r^{(i)} s^{-4} + \frac{1}{3} \left[\frac{1}{2}y'^2 + \frac{1}{2}h_{22}z'^2 - \bar{V}e^{-\sqrt{\lambda}y} - \bar{V}e^{-\sqrt{\lambda}z}\right]}{\Omega_r^{(i)} s^{-4} + \Omega_m^{(i)} s^{-3} + \frac{1}{3} \left[\frac{1}{2}y'^2 + \frac{1}{2}h_{22}z'^2 + \bar{V}e^{-\sqrt{\lambda}y} + \bar{V}e^{-\sqrt{\lambda}z}\right]}, \quad (2.10)
\end{aligned}$$

$$\frac{K}{V} = \frac{\frac{1}{2}y'^2 + \frac{1}{2}h_{22}z'^2}{\bar{V}e^{-\sqrt{\lambda}y} + \bar{V}e^{-\sqrt{\lambda}z}}. \quad (2.11)$$

For the  $\Lambda\text{CDM}$  model we have

$$\begin{aligned}
q^{\Lambda\text{CDM}} &= -\frac{\ddot{a}}{\left(\frac{\dot{a}}{a}\right)^2} = \frac{\frac{4\pi G}{3} \sum_{\alpha} (\rho_{\alpha} + 3p_{\alpha})}{8\pi G/3 \sum_{\alpha} \rho_{\alpha}} = \\
&= \frac{1}{2} \frac{[\rho_r + \rho_m + \rho_{\Lambda} + 3p_r + 3p_m + 3p_{\Lambda}]}{[\rho_r + \rho_m + \rho_{\Lambda}]} = \frac{1}{2} \frac{\left[2\Omega_r^{(i)} s^{-4} + \Omega_m^{(i)} s^{-3} - 2\Omega_{\Lambda}^{(i)}\right]}{\Omega_r^{(i)} s^{-4} + \Omega_m^{(i)} s^{-3} + \Omega_{\Lambda}^{(i)}}, \quad (2.12)
\end{aligned}$$

$$\omega_{eff}^{\Lambda\text{CDM}} = \frac{\sum_{\alpha} p_{\alpha}}{\sum_{\alpha} \rho_{\alpha}} = \frac{p_r + p_m + p_{\Lambda}}{\rho_r + \rho_m + \rho_{\Lambda}} = \frac{\frac{1}{3}\Omega_r^{(i)} s^{-4} - \Omega_{\Lambda}^{(i)}}{\Omega_r^{(i)} s^{-4} + \Omega_m^{(i)} s^{-3} + \Omega_{\Lambda}^{(i)}}. \quad (2.13)$$

### 3 Constraining and solving $\sigma\text{CDM}$ equations

We have solved background Einstein and scalar field equations of  $\sigma\text{CDM}$  model for  $\lambda$  values lying between 0.1 and 10 while keeping kinetic interaction coupling constant fixed and equal to  $\mu = 1.0$ .  $\lambda$  values are chosen as

$$\lambda = \{0.10, 0.17, 0.28, 0.46, 0.77, 1.29, 2.15, 3.59, 5.99, 10.0\}.$$

Initial contribution of matter and dark energy (chiral cosmological fields) in early epoch  $a_i = 10^{-6}$  tuned according to MCMC algorithm so that one can get current amount of dark energy today equal to 0.7 and current amount of radiation today equal to  $5 \cdot 10^{-5}$ .

From the earlier work [12] one can see that there are a substantial deviation of dark energy equation of state parameter  $\omega_{DE}$  from  $-1$  value even in model without radiation and rather late times. Our next step in this direction is finding possible nonnegligible dark energy contribution in early epochs as it take place in tracker models. So we try to take into consideration a radiation component in order to follow the more early universe evolution.

In order to fit our model to present day observations we propose target function to be minimised by MCMC procedure. We take it to be

$$\chi_{joint}^2 = \sqrt{\chi_{DE}^2 \chi_{DE}^2 + \chi_r^2 \chi_r^2},$$

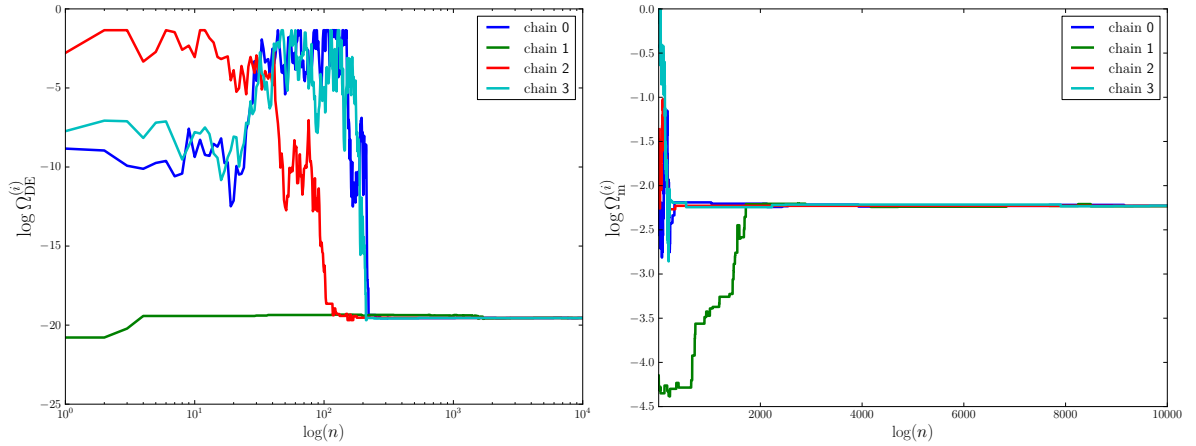
where

$$\chi_{DE}^2 = \left( \Omega_{DE}^{(0)} - \Omega_{DE}^{b.f.} \right) / \Omega_{DE}^{b.f.},$$

$$\chi_r^2 = \left( \Omega_r^{(0)} - \Omega_r^{b.f.} \right) / \Omega_r^{b.f.},$$

and  $\Omega_{DE}^{b.f.} = 0.7$ ,  $\Omega_r^{b.f.} = 5 \cdot 10^{-5}$ .

As initial contributions to critical density varies very broadly in orders we consider their decadic logarithm values.



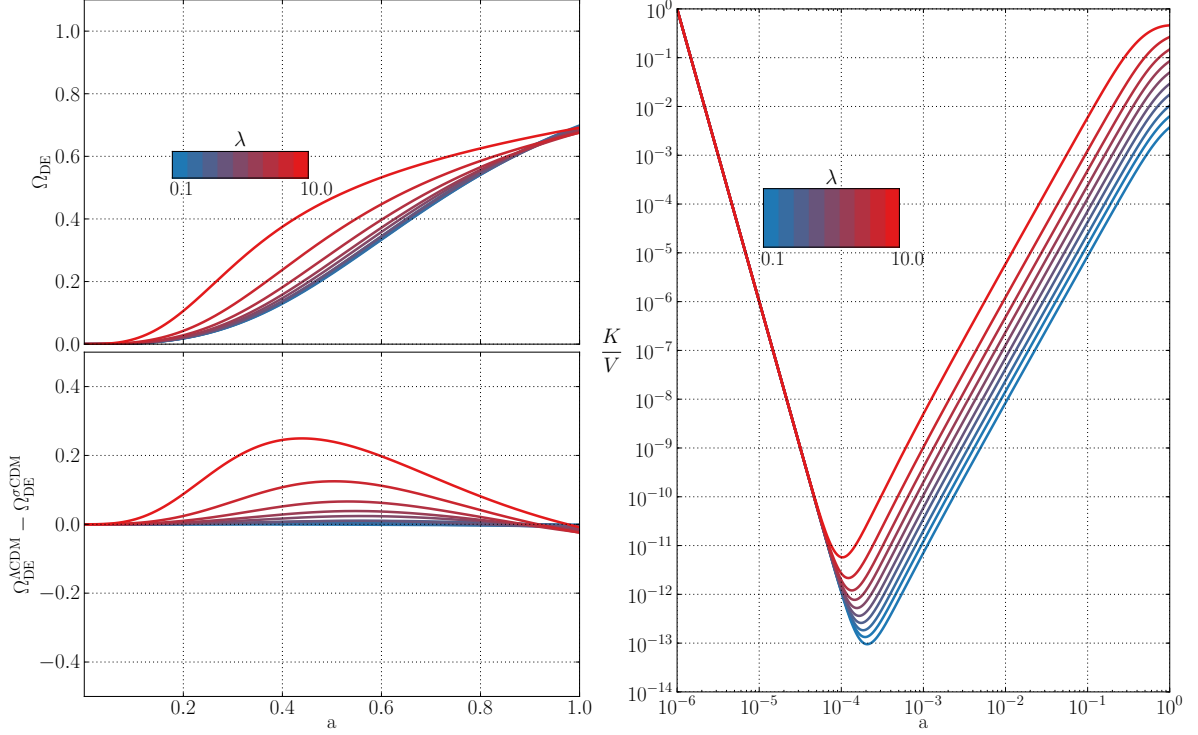
**Figure 1.** MCMC convergence demonstration for  $\Omega_{DE}^{(i)}$  and  $\Omega_m^{(i)}$  parameters in the  $\sigma$ CDM model with  $\omega_{DE}^{(i)} = 0.0$  and  $\lambda = 0.1$ .

In this paper we concentrated on case when initial value of dark energy equation of state parameter equals to 0 as in [9, 19].

## 4 Discussion

The model under investigation demonstrates rather interesting behavior when the potential interaction values is changing. As one can see from Fig. 2, the evolution of dark energy contribution to the critical density grows with increasing potential coupling constant  $\lambda$ . The smaller is the value of  $\lambda$ , the less is the deviation of  $\Omega_{DE}$  in the  $\sigma$ CDM model in comparison with the  $\Lambda$ CDM. This fact is in agreement with the general statement that if we considered limit  $\lambda \rightarrow \infty$  than we would have stiff matter behaviour described by chiral cosmological fields. Therefore we notice that for  $\lambda = 10.0$  we do not get a transition to an accelerated expansion. This is demonstrated in Fig. 3 for the deceleration parameter  $q$  and the effective equation  $\omega_{eff}$ . In the case of  $\lambda = 10.0$ , the solutions for  $q$  and  $\omega_{eff}$  do not cross 0 and  $-1/3$  values, respectively. We do not observe significant contributions of dark energy at early epochs which could suppress radiation component at previous epochs of the cosmic evolution.

It is interesting to study relation between kinetic and potential interactions in subsequent stages of cosmic evolution, see Fig. 2. An initial value of  $\omega_{DE}^{(i)} = 0$  leads to  $K = V$  at this



**Figure 2.** The left upper panel is for  $\Omega_{DE}$  evolution in  $\sigma$ CDM model and lower left panel is for comparison with  $\Omega_{DE}$  evolution in the  $\Lambda$ CDM model. The ratio of the kinetic and potential energies of chiral cosmological fields in  $\sigma$ CDM model is presented on the right while varying coupling constant in the potential in  $\sigma$ CDM

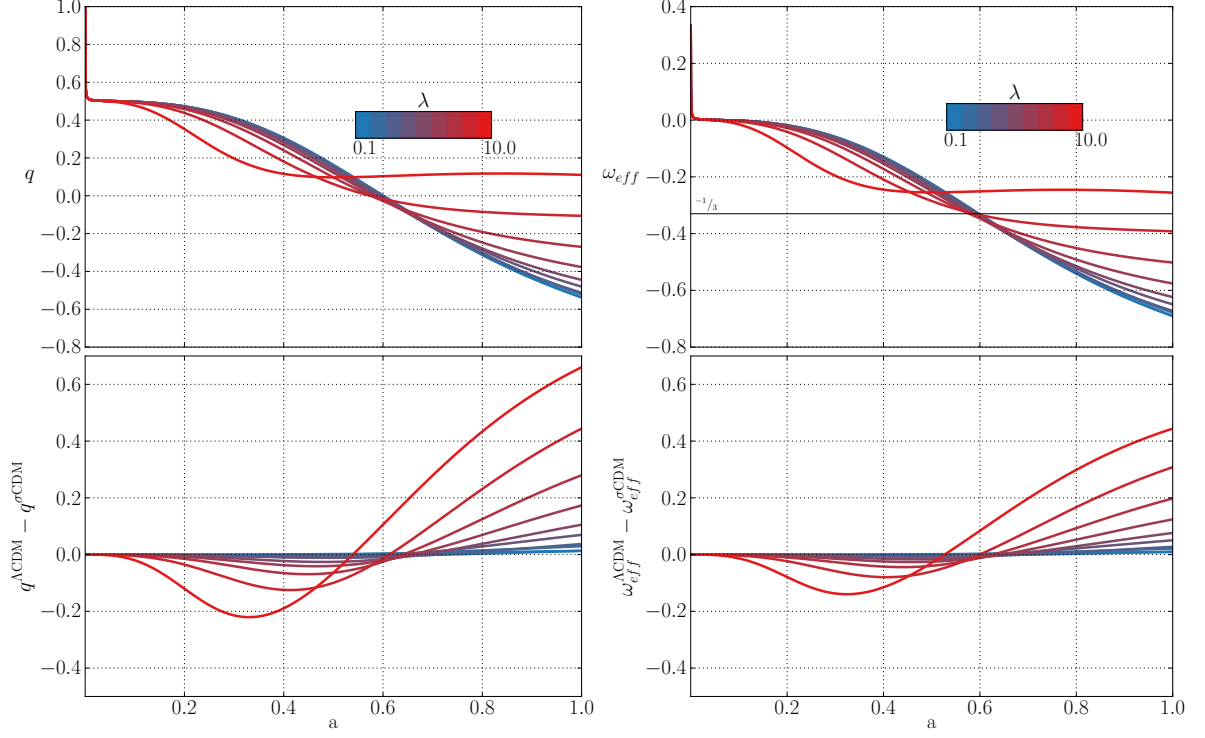
initial time. We have also found that the value  $h_{22}$  is equal to 1 almost all time and deviates somewhat in recent times, so we have a pure canonical 2-component model with very small influence from chiral metric coefficient.

We should add a few words about the behavior of dark energy equation of state  $\omega_{DE}$  described by chiral cosmological fields EoS parameter. We found that it drops down almost immediately to value of  $-1$  for all models under investigation and only at recent times there is some distinction between models with different values of  $\lambda$  as seen in Fig. 4. As we mentioned before the  $\sigma$ CDM model with  $\lambda \geq 10$  should be excluded as dark energy model, and the value  $\lambda = 5.99$  is acceptable from our criteria for viable cosmological model. Distinguishing between the remaining 9 parameters values requires additional consideration and additional observational data.

## 5 Conclusion and future work

In this article we investigated the  $\sigma$ CDM model minimally coupled with radiation. The MCMC procedure allowed us to effectively evaluate the parameters space. We have found that the kinetic interaction  $h_{22}$  has to be equal to 1 nearly all time except at the period close to the present one. To avoid this, the simple form chosen in this article should be replaced by a more complicated one. The universe described by model under consideration shows a behavior very similar to that of the  $\Lambda$ CDM model if potential interaction coupling constant  $\lambda$  is less than 1, and it deviates strongly when this parameter increases. It was found that





**Figure 3.** The deceleration parameter in the  $\sigma$ CDM in the left panels in comparison with evolution  $q$  in the  $\Lambda$ CDM model on the bottom. The effective equation of state parameter in the  $\sigma$ CDM is shown on the right in comparison with evolution  $\omega_{eff}$  in the  $\Lambda$ CDM model in the bottom right panel.

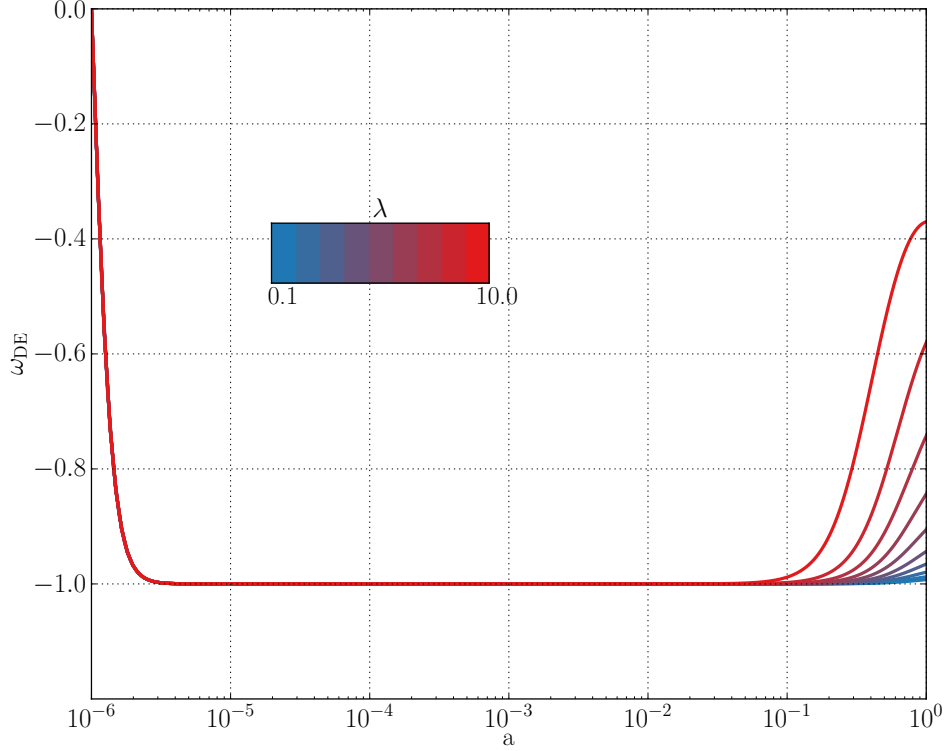
models with large  $\lambda$  does not describe an accelerated Universe expansion. This restriction may be avoided by taking into account variation of others model parameters. These issues we plan to cover in a future publication.

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**Figure 4.** The evolution of the equation of state parameter  $\omega_{DE}$  in dependence on the potential coupling constant.

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